SYSTEMATIC DESIGN OF BAND-PASS FILTER (REVISED)

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The organ of Corti consists of rows of sensory hair cells, some $7,500$ or more. Each hair cell has connections with other hair cells. A hair cell sends an electric power that is caused by the vibration of the basilar membrane at the location of the hair cell. The job of the joint of electric power lines is the subtraction of outputs from two hair cells and sends the subtracted power to the next joint. For example, assuming P_A as the output of a hair cell A and P_B as the one of a hair cell B. The subtraction $P_A - P_B$ is given by

$$R_{AB} = \begin{cases} P_A - P_B & P_A > P_B, \\ 0 & P_A < P_B, \end{cases} \tag{1}$$

u.s.w.

Similar to the previous paper the basilar membrane is assumed to function as thousands of simple band-pass filters. The normalized power spectrum of the response of a simple band-pass filter $E(f;\nu)$ is defined by a peak frequency ν and the *quality* of resonance Q, such that

$$E(f;\nu) = 1/[1 + Q^2\,(f/\nu - \nu/f)^2], \tag{2}$$

where f is a frequency.

Put

$$V_1(f;\nu) = E(f;\nu/r) - E(f;r\nu) \tag{3}$$

$$V_2(f;\nu) = E(f;r\nu) - E(f;\nu/r) \tag{4}$$

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where $r > 1$. Suppose the vibration of the membrane as a sinusoid of frequency u and magnitude M. The outputs of a hair cell specified by the frequency ν/r and $r\nu$ are given by $P_1 = ME(u;\nu/r)$ and $P_2 = ME(u;r\nu)$, respectively. Thus, the subtraction of P_1 and P_2 gives

$$R = \begin{cases} R_{12}(u) = MV_1(f;\nu) \quad u < \nu, \\ \text{or} \\ R_{21}(u) = MV_2(f;\nu) \quad u > \nu, \end{cases} \tag{5}$$

where $R_{12}(u) = 0$ for $u > v$ and $R_{21}(u) = 0$ for $u < v$.

The output of a hair cell specified by the frequency ν for the same sinusoid is given by $P_0 = ME(u;\nu)$. Thus by the subtraction of P_0 and $R_{12}(u)$, we get

$$R_0 = \begin{cases} R_{01}(u) = P_0 - R_{12}(u) \quad u < \nu, \\ R_{02}(u) = P_0 - R_{21}(u) \quad u > \nu. \end{cases} \tag{6}$$

Thus, the power spectrum of the response of a band-pass filter obtained by the processing described above is expressed as

$$B = \begin{cases} B_1(f;\nu) = E(f;\nu) - E(f;\nu/r) + E(f;r\nu) \quad f < \nu, \\ B_2(f;\nu) = E(f;\nu) - E(f;r\nu) + E(f;\nu/r) \quad f > \nu, \end{cases} \tag{7}$$

where $B(\nu;\nu) = 1$. Putting $Q = 2$ and $r = 2^{1/4} \cong 1.19$, $B(f;\nu) \cong 0$ for $f < 0.85\nu$ or $f > 1.18\nu$.

The ear detects the frequency change of 2 or 3(Hz) at the center frequency of 1(Hz). The method described in the paper[1], "The frequency discrimination of the ear" is available for this purpose, where F_A, F_p, and F_Q are given by $B(f;\nu)$, $B(f;0.86\nu)$ and $B(f;1.16\nu)$ is respectively.

References

[1] Y. Hirata, *The frequency discrimination of the ear*, http:\\wavesciencestudy.com Relevant articles, (2017)