NARROW BAND-PASS FILTER DESIGN BASED ON THE HIGHER DEGREE OF FREEDOM VIBRATION MODEL

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The design at narrow band-pass filters that might be systematically realized in the organ of Corti was presented[1], where the basilar membrane is assumed to function as thousand of simple broad band-pass filters.

The narrow band-pass filter design based on one degree of freedom vibration is simple and understandable. It is, however, more likely to explain the narrow band-pass filter design based on the higher degree of freedom vibration that may simulate the vibration of the basilar membrane. Similar to the one degree of freedom vibration model, number of subtraction of electric power signals from hair cells are assumed in the organ of Corti.

The organ of Corti consists of rows of sensory hair cells, some $7,500$ or more. Each hair cell has connections with sensory hair cells. A hair cell sends an electric power that is caused by the vibration of the basilar membrane at the location of the hair cell. At the joint of electric power lines the subtraction detects the difference of powers sent from hair cells. Suppose P_A is the output of a hair cell A and P_B that of a hair cell B. The subtraction, $Sub(P_A, P_B)$, is given by

$$(1) \qquad Sub(P_A, P_B) = \begin{cases} P_A - P_B & P_A > P_B, \\ 0 & P_A = P_B, \\ P_B - P_A & P_B > P_A. \end{cases}$$

The subtraction is similar to the all wave rectification of electric circuits.

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Put

$$S(f;\nu_m) = 1/Q_m^2 + (f/\nu_m - \nu_m/f)^2\,, \tag{2}$$

where f is a frequency. The power spectrum of the frequency response of one degree of freedom vibration is expressed by $1/S(f;\nu_m)$. The *quality* of resonance is given by Q_m and a resonance frequency by ν_m. In general, the power frequency response of n-degree-of-freedom vibration is expressed by

$$E_n(f;\nu) = \frac{\prod_{m=1}^{n} S(f;\nu_{2m-2})}{\prod_{m=1}^{n} S(f;\nu_{2m-1})}, \tag{3}$$

where $\nu = \nu_1, S(f;\nu_0) = 1$ and $\nu_1 < \nu_2 < \cdots < \nu_{2n-1}$. Figure 1 shows the responses $E_n(f;\nu)$ where $n = 3, \nu_m = m\nu$ and $Q_m = Q$ (Q : 1.0, 1.4, and 2.0).

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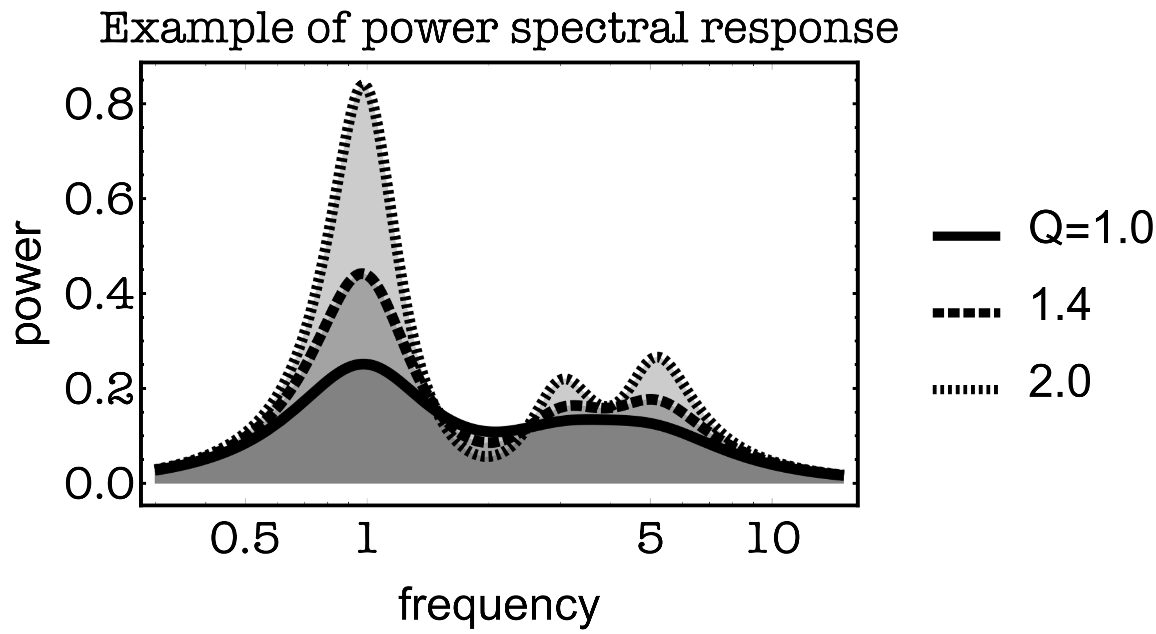

FIGURE 1

Consider three hair cells whose locations on the basilar membrane are specified by the frequencies $\nu, \nu/r_1,$ and $r_2\nu$ and the power frequency responses given by

$E_n(f;\nu), E_n(f;\nu/r_1)$, and $E_n(f;r_2\nu)$, respectively, where $1 < r_1,\ r_2 < 2$. The parameters r_1 and r_2 satisfy

$$E_n(f;\nu/r_1) = E_n(f;r_2\nu) \tag{4}$$

at $f = \nu_c$ where $\nu_c \cong \nu$. In the case of one-degree-of-freedom vibration model, $\nu_c = \nu$ and $r_1 = r_2$.

If we put the power outputs of these three hair cells such that

$$\begin{aligned} P_0 &= E_n(f;\nu) \\ P_1 &= E_n(f;\nu/r_1) \\ &\text{and} \\ P_2 &= E_n(f;r_2\nu), \end{aligned} \tag{5}$$

the subtraction of P_1 and P_2 is given by,

$$Sub(P_1, P_2) = \begin{cases} P_1 - P_2 & f < \nu_c, \\ 0 & f = \nu_c, \\ P_2 - P_1 & f > \nu_c. \end{cases} \tag{6}$$

Thus we get the power spectrum at a narrow band-pass filter by the difference $D(P_0, S)$ that is given by subtracting $Sub(P_1, P_2)$ from P_0, such that, for $P_0 > Sub(P_1, P_2)$,

$$D(P_0, S) = \begin{cases} P_0 - P_1 + P_2 & f < \nu_c, \\ P_0 & f = \nu_c, \\ P_0 - P_2 + P_1 & f > \nu_c, \end{cases} \tag{7}$$

or, if $P_0 < Sub(P_1, P_2)$ for $f < f_1$ or $f > f_2$,

$$D(P_0, S) = \begin{cases} 0 & f < f_1, \\ P_0 - P_1 + P_2 & f_1 < f < \nu_c, \\ P_0 & f = \nu_c, \\ P_0 - P_2 + P_1 & \nu_c < f < f_2, \\ 0 & f > f_2, \end{cases} \tag{8}$$

where the negative values are set to zero.

Examples of narrow band-pass filters given by Eqs.7 and 8 are shown in Fig.2 and 3 where $n = 3$ and $Q = 2$. The parameters are $r_1 = r_2 = 2^{1/6}$ in Fig.2, and

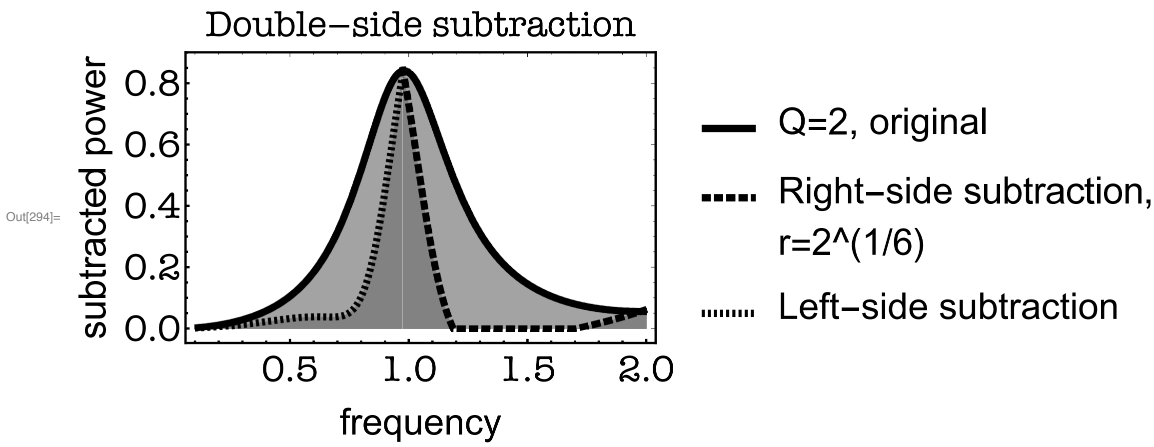

FIGURE 2

$r_1 = r_2 = 2^{1/4}$ are for Fig.3.

A power spectrum filter (PSF) represented by the band-pass filter described above involves the non-linear processing of the subtractions that makes differences between the PSF and a usual electric filter. Suppose the basilar membrane is excited by the vibration of a random noise whose time-varying power spectrum is given by $x(f, t)$ where t is a time and $a < f < b$, so that the outputs of the three hair

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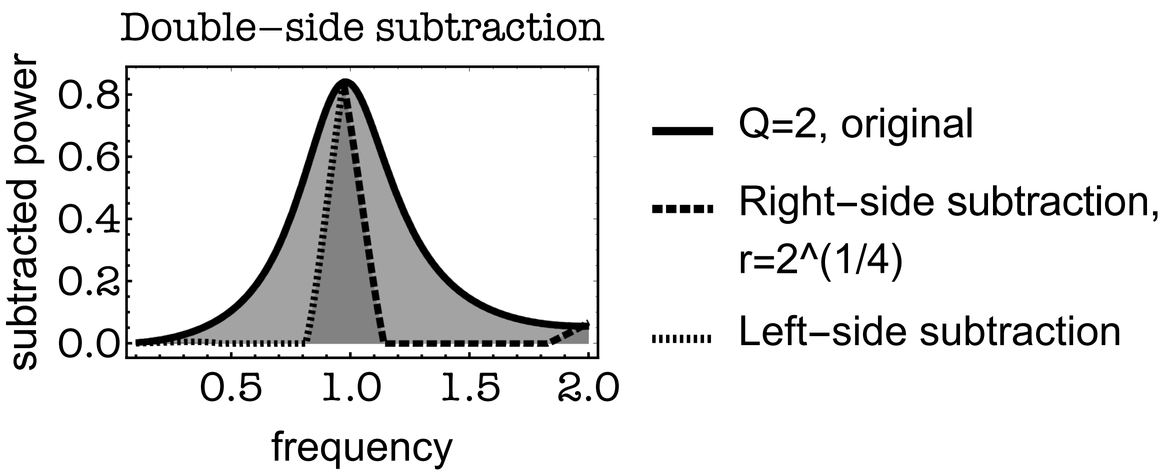

FIGURE 3

cells are

$$P_0 = R_0(t) = \int_a^b x(f;t)E_n(f;\nu)df \tag{9}$$

$$P_1 = R_1(t) = \int_a^b x(f;t)E_n(f;\nu/r_1)df$$

and

$$P_2 = R_2(t) = \int_a^b x(f;t)E_n(f;\nu/r_2)df. \tag{10}$$

If $a < f_1,\ b > f_2$ and $R_1(t) \cong R_2(t)$, then, $P_1 \cong P_2$ at time t, we have $D(P,S) \cong P_0 = R_0(t)$. This suggests that the response at the band-pass filter realized in the organ of Corti changes adaptively with the input signal or noise, namely, a narrow band for a sinusoidal and/or a broad band for a random noise.

The ear can not detect the change of sound intensity less than 1(dB). Applying the fact to the magnitude near the peak frequency of the band-pass filter shown in Fig.3, the detectable frequency change is roughly given by 0.03ν. The ear detects the frequency change of 2 or 3(Hz) at the center frequency of 1(kHz). Details account for the frequency discrimination of the ear is shown in the reference [2], where F_A, F_P and F_Q are given by the method described above.

References

[1] Y. Hirata, *Systematic design of band-pass filter (revised)*, http:\\wavesciencestudy.com Relevant articles, (2017)

[2] Y. Hirata, *The frequency discrimination of the ear*, http:\\wavesciencestudy.com Relevant articles, (2017)